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DYNAMIC MODELL ADEQUATE FOR ROBOT CONTROL

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Abstract: The dynamic model of the robot can be obtained from the known physical laws: the d′Alembert principle, the Lagrangian formulation or the Newton-Euler equations. These motion equations are equivalent to each other in the sense that they describe the dynamic behaviour of the same physical robot. However the structure of these equations may differ. In this paper the computational effort for a point on the trajectory in each of the three formulations is presented. It is shown that for an n DOF robot, the computational time effort is of different time orders. Also the control model suitable for robots which move at slow speeds is presented.

1. INTRODUCTION

The dynamic performance of the robots depends on the efficiency of the control algorithms and the dynamic model of the robot. In order to solve the control problem, first is necessary to obtain a dynamic model describing very exactly the behavior of the robot and then specifying corresponding control laws on strategies to achieve the desired system response and performance. So the purpose of the robot control is to maintain the dynamic response of the computer-based robot in accordance with the specified system performance and desired goals [1,6].

2. THE DYNAMIC MODEL

The dynamic equations of motion of the serial and parallel robots is a set of mathematical equations describing the dynamic behavior of the robot, that are useful for simulation of the end-effector motion and also suitable for the design of the control equation of the manipulated object [2,3,4].

The basic geometric structure of the 6-PGK parallel robot developed at "Petru Maior" University of Târgu-Mureş consists of a mobile platform (supporting the end-effector) connected to the adjacent links at six distinct points at the level III by cardanic kinematic pairs. Six legs connect the platform to the base. Each leg has an actuated translational kinematic pair at level I and a spherical kinematic pair at level II. The location (position and orientation) of the end-effector and of the manipulated object are expressed by the generalized coordinates of the manipulated object denoted q_{p_i} (i=1,2,...,6). The displacements in the actuated translational kinematic pairs (actuator displacements) are the generalized coordinates of the parallel robot q_i (i=1,2,...,6).

In order to develop the motion equations of the manipulated object, the d′Alembert principle, the Lagrangian formulation or the Newton-Euler equations can be employed.

The Lagrangian formulation describes the behavior of the robot in terms of work and energy stored in the system rather than in terms of forces and moments of the individual members involved. So deduction of the dynamic model of the robot by using the Lagrange formulation is simple, but the computation of the dynamic coefficients requires a fair amount of arithmetic operations and the motion equations are very difficult to utilize in real-time control.

Each trajectory point of a robot with n degrees of freedom, needs the following computational effort (M = multiplications, A = additions):

- $M = \frac{128}{3}n^4 + \frac{512}{3}n^3 + \frac{739}{3}n^2 + \frac{160}{3}n$,
- $A = \frac{98}{3}n^4 + \frac{781}{6}n^3 + \frac{559}{3}n^2 + \frac{245}{6}n$.

An alternative in deriving equations of motion is the d′Alembert principle. The equations are expressed in matrix form suitable for control analysis. Also, the contribution of the rotational and translational effects of the links is explicitly identified, that is very useful for designing a controller in state space. The computational effort for each trajectory point in this method is:

- $M = \frac{13}{6}n^3 + \frac{105}{2}n^2 + \frac{268}{3}n + 69$,
- $A = \frac{4}{3}n^3 + 44n^2 + \frac{146}{3}n + 45$.

Another approach for obtaining a very efficient set of equations of motion is the computation of the generalized force-torque vector with the Newton–Euler equations. This set of recursive equations can be applied to the serial architectures. The main advantage of this formulation is that the time computation of the generalized force-torque vector is linearly proportional to the number of degrees and independent to the robot arm configuration. This formulation is suitable for real-time control in the joint-variable space.

The computational effort expressed in number of multiplications and additions for each trajectory point is:

- $M = 132n$,
- $A = 111n \quad 4$.

In conclusion, for a robot with n degrees of freedom the control law based on the Lagrange formulation can be computed in $O(n^4)$ time. The analogous control law derived from the d′Alembert principle can be computed in $O(n^3)$ time and the generalized force-torque vector with the Newton–Euler equations can be computed in $O(n)$ time.

3. CONTROLLER FOR ROBOT

The computed torque technique is a feed forward control, which has feedforward and feedback components. The necessary correction torque to compensate from any deviations on the desired trajectory is computed by the feedback component.

Let $q_1(t)$, $q_2(t), \ldots, q_n(t)$ be the generalized coordinates of the robot and $\tau_1(t)$, $\tau_2(t), \ldots, \tau_n(t)$ the corresponding generalized force to drive the robots legs.

The equations of motion of the 6-PGK parallel robot are [2]:

$$\frac{1}{2}\sum_{k=1}^{3}\sum_{l=1}^{3}\dot{J}_{kl} \quad \frac{\partial \omega_k}{\partial \dot{q}_{p_j}}\omega_l + \omega_k \frac{\partial \omega_l}{\partial \dot{q}_{p_j}} \qquad \frac{1}{2}\sum_{k=1}^{3}\sum_{l=1}^{3}\frac{\partial J_{kl}}{\partial q_{p_j}}\omega_k\omega_l +$$

$$\frac{1}{2}\sum_{k=1}^{3}\sum_{l=1}^{3}J_{kl}\quad\frac{d}{dt}\quad\frac{\partial\omega_k}{\partial\dot{q}_{p_j}}\quad\omega_l+\frac{\partial\omega_k}{\partial\dot{q}_{p_j}}\varepsilon_l+\varepsilon_k\frac{\partial\omega_l}{\partial\dot{q}_{p_j}}+\omega_k\frac{d}{dt}\quad\frac{\partial\omega_l}{\partial\dot{q}_{p_j}}\quad\omega_l\quad+$$

$$M\sum_{k=1}^{3}\ddot{q}_{p_k}\frac{\partial\dot{q}_{p_k}}{\partial\dot{q}_{p_j}}+\sum_{i=1}^{6}m_i\quad\ddot{q}_i\frac{\partial\dot{q}_i}{\partial\dot{q}_{p_j}}+\dot{q}_i\frac{d}{dt}\quad\frac{\partial\dot{q}_i}{\partial\dot{q}_{p_j}}+\qquad\frac{1}{2}\sum_{k=1}^{3}\sum_{l=1}^{3}J_{kl}\quad\frac{\partial\omega_k}{\partial q_{p_j}}\omega_l+\omega_k\frac{\partial\omega_l}{\partial q_{p_j}}\qquad\sum_{i=1}^{6}m_i\dot{q}_i\frac{\partial\dot{q}_i}{\partial q_{p_j}}=\tau_j$$

$$,(j=1,2,...,6)\tag{1}$$

The dynamic model of the robot (1) is in the form:

$$[M(q)]\{\ddot{q}(t)\}+\{C(q,\dot{q})\}\{\dot{q}(t)\}+\{G(q)\}=\{\tau(t)\}\tag{2}$$

where

$[M(q)]$ is the $n\times n$ inertial matrix;

$\{C(q,\dot{q})\}$ is the $n\times 1$ nonlinear Coriolis and centrifugal force vector;

$\{G(q)\}$ is the gravity loading force vector;

$\{\tau(t)\}^T=\{\tau_1(t),\tau_2(t),\ldots,\tau_n(t)\}$ is the generalized force vector;

$\{q(t)\}=\{q_1(t),q_2(t),\ldots,q_n(t)\}$ is the generalized coordinates vector;

$\{\dot{q}(t)\}=\{\dot{q}_1(t),\dot{q}_2(t),\ldots,\dot{q}_n(t)\}$ is the velocities coordinates vector;

$\{\ddot{q}(t)\}=\{\ddot{q}_1(t),\ddot{q}_2(t),\ldots,\ddot{q}_n(t)\}$ is the generalized coordinates vector.

In the computed torque technique one can compute the counterparts of $[M(q)]$, $\{C(q,\dot{q})\}$ and $\{G(q)\}$ in the equation of motion (2), in order to minimize their nonlinear effects, and use a proportional plus derivative control to servo the motor kinematics pairs.

The control law is in the form:

$$[M(q)]\;\left\langle\{\ddot{q}(t)\}^d+[K_v]\left\langle\{\dot{q}(t)\}^d\quad\{\dot{q}(t)\}\right\rangle+[K_p]\left\langle\{q(t)\}^d\quad\{q(t)\}\right\rangle\right\rangle+\{C(q,\dot{q})\}\;\{\dot{q}(t)\}+\{G(q)\}\;=\{\tau(t)\}\tag{3}$$

where $[K_v]$ is the $n\times n$ derivative feedback gain matrix and $[K_p]$ is the $n\times n$ position feedback gain matrix [5].

From equations (1) and (2) we deduce:

$$[M(q)]\{\ddot{q}(t)\}+\{C(q,\dot{q})\}\{\dot{q}(t)\}+\{G(q)\}=$$

$$=[M(q)]\;\left\langle\{\ddot{q}(t)\}^d+[K_v]\left\langle\{\dot{q}(t)\}^d\quad\{\dot{q}(t)\}\right\rangle+[K_p]\left\langle\{q(t)\}^d\quad\{q(t)\}\right\rangle\right\rangle+\{C(q,\dot{q})\}\;\{\dot{q}(t)\}+\{G(q)\}\;.\tag{4}$$

If $[M(q)]=[M(q)]$, $\{C(q,\dot{q})\}=\{C(q,\dot{q})\}$, $\{G(q)\}=\{G(q)\}$ and we denote the position error vector with $\{e(t)\}=\{q(t)\}^d\quad\{q(t)\}$, than equation (4) becomes:

$$[M(q)]\left\langle\{\ddot{e}(t)\}+[K_v]\{\dot{e}(t)\}+[K_p]\{e(t)\}\right\rangle=0\,.\tag{5}$$

The matrices $[K_v]$ and $[K_p]$ can be chosen appropriately since the inertial matrix is always nonsingular. So the characteristic roots of equation (5) have negative real parts and the position error approaches zero asymptotically.

If the robot is moving at low speeds, by neglecting the velocity – related coupling term $\{C(q,\dot{q})\}$ and the off-diagonal elements of the inertial matrix from equation (3), we deduce the control law:

$$[M(q)]\ \left\langle\{\ddot{q}(t)\}^d + [K_v]\left\langle\{\dot{q}(t)\}^d \quad \{\dot{q}(t)\}\right\rangle + [K_p]\left\langle\{q(t)\}^d \quad \{q(t)\}\right\rangle\right\rangle + \{G(q)\} = \{\tau(t)\} \tag{6}$$

where in the left side of the equation, we have a diagonal matrix. Robots controlled in this manner move at low speeds with unnecessary vibrations.

In the joint-variable space an analogous control law can be obtained recursively from the Newton-Euler equations of motion. So the desired generalized coordinates of the robot are deduced in the form:

$$\ddot{q}_i(t) = \ddot{q}_i^d(t) + \sum_{j=1}^{n} \dot{e}_j(t)K_v^{ij} + \sum_{j=1}^{n} e_j(t)K_p^{ij}\ddot{q}_i^d(t) + \sum_{j=1}^{n}\left\langle \dot{q}_j^d(t) \quad \dot{q}_j(t)\right\rangle K_v^{ij} + \sum_{j=1}^{n}\left\langle q_j^d(t) \quad q_j(t)\right\rangle K_p^{ij} \tag{7}$$

where $e_j(t)$ is the position error for joint j, K_v^{ij} and K_p^{ij} are the derivative and the position feedback gains for joint i.

The control law is proportional plus derivative and has the effect of compensating for inertial loading, coupling effects and gravity loading of the links. In order to achieve a critical damped system for each motor kinematics pair subsystem the feedback gain matrices can be chosen.

Based on the Lagrange equation of motion, the generalized torque-force vector can be computed in $O(n^4)$ time, and the analogous control law derived from the Newton-Euler equations of motion can be computed in $O(n)$ time.

The main drawback of this control technique is that the convergence of the position error vector depends on the dynamic coefficients in the equation of motion (2). Any significant performance gain in this and other areas of end-effector control require the consideration of more efficient dynamic models, sophisticated control approaches, and the use of dedicated computer architectures and parallel processing techniques.

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